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A spatial Solow model with transport cost

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Abstract

In this paper we introduce capital transport cost in an unidimensional unbounded economy described by a spatial Solow model with capital-induced labor migration. Proceeding with a linear stability analysis of its spatially homogeneous equilibrium solution, we show that exists a critical value for the capital transport cost where the dynamic behavior of the economy changes, provided the capital-induced labor migration intensity is big enough. On one hand, if capital transport cost is bigger than this critical value, the homogeneous equilibrium of the model is stable, and the economy converges to this spatially homogeneous state in the long run; on the other hand, if transport cost is smaller than this critical value, the equilibrium is unstable, and the economy may develop distinct spatio-temporal dynamics, including the formation of stable economic clusters and spatio-temporal economic cycles, depending on the other parameters of the model. This result, though obtained using a different formalism, is consistent with the main results of the standard core-periphery model used in the New Economic Geography literature, where a small transport cost is essencial to the formation of spatial economic agglomeration. Finally, we close this work validating the linear stability analysis results through numerical simulations, and verifying that the introduction of a positive transport cost in the model causes a break in the symmetry of the spatial economic agglomerations generated.

JEL Classification: R12, O40.

Keywords: Spatial Solow Model, Regional Science, Economic Agglomeration, Economic Geography.

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1 Introduction

In the literature we can find two main fields of study considering, in an explicit way, the spatial structure of an economic system in its models. The older one, beginning in the 1950s, is Regional Science (RS), which follows a macroeconomic approach. Considering local spatial interactions between capital and labor force, RS solves a variety of spatial economic growth models, aiming to find the optimal space-time distribution of product which maximise a certain social utility function (Isard, 1956; Isard and Liosatos, 1975a,b, 1979; Isard, 2003). The other one, New Economic Geography (NEG), began with Krugman in the 1990s (Krugman, 1991), and was developed in great detail in Fujita et al. (1999), Fujita and Thisse (2002), Combes et al. (2008) and Brakman et al. (2009), among others. NEG, which is a spatial generalization of the New Trade Theory proposed by Krugman approximately a decade earlier (Krugman, 1979), deals with microeconomic models and has the core-periphery model as its standard model. One of the main implication of the core-periphery model is that the combination of a lower transport cost, a mobile labor force and the presence of increasing returns can result in spatial agglomeration of economic activity.

Perhaps because of the success obtained by NEG among economists, Regional Science models for spatial economic growth had reappeared in the literature since the 2000s. Quigley, for example, talks about a rebirth of regional sciences (Quigley, 2001). Beginning with Camacho and Zou (2004), a series of spatial Solow models were presented, considering a continuous space-time framework, all of which can be considered particular cases of a general model presented earlier by Isard and Liosatos (1979). Examples of these works are Brito (2005), Engbers (2009), Capasso et al. (2010) and Capasso et al. (2012). Other spatial models analysed include a spatial version of the Ramsey model (Brito, 2004; Camacho et al., 2008), and one of the AK model (Boucekkine et al., 2013). It is interesting to note that, of all these spatial models, only the spatial version of the Ramsey model considered in Brito (2004) was able to develop, endogenously, economic agglomerations.

More recently, considering an economy where all locations use the same Cobb-Douglas production function, whose labor force grows following a logistic equation, and where both capital and labor follow the principle of diminishing marginal return, diffusing to regions where they are more scarce, Juchem Neto and Claeysen (2014) showed that the introduction of a capital-induced labor migration in such spatial Solow model is a necessary condition to endogenously generate economic agglomeration and cycles. Capital-induced labor migration is defined as a migratory behavior where workers move from regions with a lower density of capital to regions with higher density of capital.

In this work we shall introduce capital transport cost in the spatial Solow model presented in Juchem Neto and Claeysen (2014). We also will consider, for analytical simplicity, an unbounded and unidimensional spatial structure for the economy. With this framework we shall show that if the capital transport cost is too high, this modified model makes the economy converge to a spatially homogeneous equilibrium, with no agglomeration formation. On the other hand, if transport cost is lower than a critical value, the economy may develop capital and labor agglomerations, or spatio-temporal cycles, depending on the parameters of the system. We obtain such a critical value for the transport cost via a standard linear stability analysis of the spatially homogeneous

equilibrium points (or steady-states) of the system (Zhu and Murray, 1995; Murray, 2003; Edelstein-Keshet, 2005). This result, though obtained using a different formalism and under a macroeconomic framework, is consistent with the behavior of the standard core-periphery model widely used in NEG.

This paper is structured as follows. In Section 2 we present our model and in Section 3 we carried out a linear stability analysis of its spatially homogeneous equilibrium point, deriving conditions that make the model operates in an unstable regime where the formation of economic agglomeration and cycles is possible. Following, in Section 4 we validate the previous linear stability analysis results through numerical simulations, which we also use to illustrate the types of spatio-temporal dynamics developed by the model. Finally, we close with conclusions and perspectives for future works.

2 The Model

In this model we consider a continuum of local economies described by the real line $\Omega = \mathbb{R}$. Each point in this continuum $x \in \Omega$ is a local producing unity endowed with capital and labor densities, $K(t, x) \geq 0$ and $L(t, x) \geq 0$, respectively. These factors are used in the production of an aggregated good, $Y(t, x) \geq 0$, through a Cobb-Douglas production function, $Y(t, x) = AK^\phi L^{1-\phi}$. Here $A > 0$ is a constant technological parameter and $\phi \in (0, 1)$ gives the capital share used in the production. Besides of that, we suppose the labor force organic growth is governed by a logistic equation, so it is bounded in the long run. Note that space is homogeneous, in the sense that each locality uses the same production function and has a labor growth given by the same logistic law. Finally, it is also important to note that, for simplicity, we consider only one-directional trade, namely in the positive x-direction (see chapter 5 of Isard and Liossatos (1979)).

The derivation of the model follows closely the one presented in Juchem Neto and Claeysen (2014), so we referred the reader to that work. The only difference is the introduction of capital transport cost in the continuity equation for the capital, which follows from the balance of capital in the economy:

$$\frac{\partial K}{\partial t} = h - \frac{\partial \tau}{\partial x} - \rho_K \tau. \quad (1)$$

In this equation, $h(K, L) = sAK^\phi L^{1-\phi} - \delta K$ is the net investment, where $s \in (0, 1)$ is the saving rate and $\delta \in (0, 1)$ is the capital depreciation rate; $\tau(t, x)$ is the flux of capital passing through location x at time t ; and $\rho_K \in [0, 1)$ is the capital transport cost rate, in physical terms. Following Isard and Liossatos (1979), we assume that the capital transport cost, $\rho_K \tau$, is paid by each location $x \in \Omega$ and is a physical fraction of the flux of capital flowing through that location¹.

For the flux of capital $\tau(t, x)$, we consider that it moves from regions with higher density of capital to regions with a lower density of capital (what obeys the neoclassical principle of diminishing marginal return of capital):

$$\tau = -d_K \frac{\partial K}{\partial x} \quad (2)$$

¹In this sense, this is similar to the widely used iceberg costs in economics literature.

where $d_K \geq 0$ is the capital diffusion coefficient. Then, plugging (2) into (1) we get the equation governing the evolution of the distribution of capital in the economy:

$$\frac{\partial K}{\partial t} = h + d_K \frac{\partial^2 K}{\partial x^2} + \rho_K d_K \frac{\partial K}{\partial x}. \quad (3)$$

The continuity equation for the labor force, in its turn, is kept unchanged, and is given by:

$$\frac{\partial L}{\partial t} = g - \frac{\partial H}{\partial x}, \quad (4)$$

where $g(K, L) = g(L) = aL - bL^2$, with the population growing rate $a > 0$, and $\frac{a}{b}$ the maximum labor capacity of each local economy. For the flux of labor, $H(t, x)$, we consider, on one hand, that labor moves from regions of high density of workers to regions with low density of workers (diminishing marginal return of labor) and, on the other hand, that it also moves into regions with a higher density of capital, what we call capital-induced labor migration:

$$H(t, x) = -d_L \frac{\partial L}{\partial x} + \chi_L L \frac{\partial K}{\partial x}. \quad (5)$$

Combining equations (4) and (5), we obtain the equation governing the distribution of labor force in the economy:

$$\frac{\partial L}{\partial t} = g + d_L \frac{\partial^2 L}{\partial x^2} - \chi_L \frac{\partial}{\partial x} \left(L \frac{\partial K}{\partial x} \right), \quad (6)$$

where $d_L \geq 0$ is labor diffusion coefficient, and $\chi_L \geq 0$ is the capital-induced labor migration coefficient.

To complete the model, we consider nonnegative initial distributions of capital and labor, $K(0, x) = K_0(x)$ and $L(0, x) = L_0(x)$, $x \in \mathbb{R}$, and impose homogeneous Neumann conditions on infinity, i.e., $\lim_{|x| \rightarrow \infty} \frac{\partial K}{\partial x} = \lim_{|x| \rightarrow \infty} \frac{\partial L}{\partial x} = 0$. That means that there is no flux of capital and labor far away from the origin. Putting all these pieces together, our model is given by the following system of coupled partial differential equations, with reaction, diffusion and advection terms:

$$\frac{\partial K}{\partial t} = h(K, L) + d_K \frac{\partial^2 K}{\partial x^2} + \rho_K d_K \frac{\partial K}{\partial x}, \quad x \in \mathbb{R}, \quad t > 0 \quad (7a)$$

$$\frac{\partial L}{\partial t} = g(K, L) + d_L \frac{\partial^2 L}{\partial x^2} - \chi_L \frac{\partial}{\partial x} \left(L \frac{\partial K}{\partial x} \right), \quad x \in \mathbb{R}, \quad t > 0 \quad (7b)$$

$$K(0, x) = K_0(x), \quad L(0, x) = L_0(x), \quad x \in \mathbb{R} \quad (7c)$$

$$\lim_{|x| \rightarrow \infty} \frac{\partial K}{\partial x}(t, x) = \lim_{|x| \rightarrow \infty} \frac{\partial L}{\partial x}(t, x) = 0, \quad t \geq 0 \quad (7d)$$

Comparing this system with the model analysed in Juchem Neto and Claeysen (2014), the difference is the term $\rho_K d_K \frac{\partial K}{\partial x}$, which models capital transport cost and is the contribution of the present work. Note that if we make $\rho_K = 0$, we recover that previous model. Also follows directly from Proposition 1 of the cited work the fact that, if we have smooth enough nonnegative initial distributions of capital and labor, $K_0(x)$ and $L_0(x)$, then the classical solutions of (7a)-(7d) are always nonnegative. Therefore, the model always generates economically realistic solutions.

3 Linear Stability Analysis

We start this section noting that if we make $h(K, L) = 0$ and $g(K, L) = 0$, we can find the non-trivial spatially homogeneous equilibrium solutions of the system (7a)-(7d), which are given by $K_\infty = \frac{a}{b} \left(\frac{sA}{\delta} \right)^{\frac{1}{1-\phi}}$ and $L_\infty = \frac{a}{b}$.

Before proceeding with the linear stability analysis, it is algebraically convenient to rewrite (7a)-(7d) in an adimensional form. Then, considering the re-scaled variables:

$$K^* = \frac{K}{K_\infty}, \quad L^* = \frac{L}{L_\infty}, \quad t^* = at, \quad x^* = \sqrt{\frac{a}{d_K}} x, \quad (8)$$

we can rewrite (7a)-(7d) in adimensional form, where we dropped the asterisks in order to keep notation simple:

$$\frac{\partial K}{\partial t} = h(K, L) + \frac{\partial^2 K}{\partial x^2} + \rho \frac{\partial K}{\partial x}, \quad x \in \mathbb{R}, \quad t > 0 \quad (9a)$$

$$\frac{\partial L}{\partial t} = g(K, L) + d \frac{\partial^2 L}{\partial x^2} - \chi \frac{\partial}{\partial x} \left(L \frac{\partial K}{\partial x} \right), \quad x \in \mathbb{R}, \quad t > 0 \quad (9b)$$

$$K(0, x) = K_0(x), \quad L(0, x) = L_0(x), \quad x \in \mathbb{R} \quad (9c)$$

$$\lim_{|x| \rightarrow \infty} \frac{\partial K}{\partial x}(t, x) = \lim_{|x| \rightarrow \infty} \frac{\partial L}{\partial x}(t, x) = 0, \quad t \geq 0 \quad (9d)$$

where, from now on:

$$h(K, L) = \beta(K^\phi L^{1-\phi} - K) \\ g(K, L) = g(L) = L(1 - L)$$

and the new adimensional parameters are given by $\beta = \frac{\delta}{a}$, $d = \frac{d_L}{d_K}$, $\chi = \frac{a}{b} \frac{\chi_L}{d_K} \left(\frac{sA}{\delta} \right)^{\frac{1}{1-\phi}}$ and $\rho = \rho_K \sqrt{\frac{d_K}{a}}$. In this formulation, the non-trivial spatially homogeneous equilibrium solutions of the model turns out to be normalized, i.e., $K_\infty = L_\infty = 1$.

Defining a spatially non-homogeneous small amplitude perturbation of the capital and labor equilibrium, $u_K = K - K_\infty$ and $u_L = L - L_\infty$, respectively, we can linearize (9a) and (9b) around their equilibrium points K_∞ and L_∞ . Using Taylor Theorem and keeping only linear terms we get:

$$\frac{\partial u_K}{\partial t} = -\beta(1 - \phi)u_K + \beta(1 - \phi)u_L + \frac{\partial^2 u_K}{\partial x^2} + \rho \frac{\partial u_K}{\partial x}, \quad x \in \mathbb{R}, \quad t > 0 \quad (10a)$$

$$\frac{\partial u_L}{\partial t} = -u_L + d \frac{\partial^2 u_L}{\partial x^2} - \chi \frac{\partial^2 u_K}{\partial x^2}, \quad x \in \mathbb{R}, \quad t > 0 \quad (10b)$$

$$K(0, x) = K_0(x), \quad L(0, x) = L_0(x), \quad x \in \mathbb{R} \quad (10c)$$

$$\lim_{|x| \rightarrow \infty} \frac{\partial K}{\partial x}(t, x) = \lim_{|x| \rightarrow \infty} \frac{\partial L}{\partial x}(t, x) = 0, \quad t \geq 0 \quad (10d)$$

Now, looking for solutions for the linearized system in the form $u_K = C e^{\sigma t + i k x}$ and $u_L = D e^{\sigma t + i k x}$, we get the following linear system:

$$\begin{bmatrix} (\sigma + \beta(1 - \phi) + k^2 - i \rho k) & -\beta(1 - \phi) \\ -\chi k^2 & (\sigma + d k^2 + 1) \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (11)$$

where $i = \sqrt{-1}$ and σ is the growth rate of the mode with wavenumber k . This linear system (11) admits non-trivial solution if and only if \mathbf{M} is singular, that is, if:

$$P(\sigma) = \det \mathbf{M} = (\sigma + \beta(1 - \phi) + k^2 - ik\rho)(\sigma + dk^2 + 1) - \beta(1 - \phi)\chi k^2 = 0, \quad (12)$$

which can be rewritten as:

$$P(\sigma) = \sigma^2 + \hat{z}\sigma + \hat{w} = 0 \quad (13)$$

where:

$$\hat{z} = z + ib = [(1 + d)k^2 + \beta(1 - \phi) + 1] + i(-\rho k) \quad (14a)$$

$$\hat{w} = w + ic = \{dk^4 + [1 + \beta(1 - \phi)(d - \chi)]k^2 + \beta(1 - \phi)\} + i[-\rho k(dk^2 + 1)] \quad (14b)$$

Note that the parameter ρ , which is directly proportional to the original capital transport cost, ρ_K , appears only in the imaginary parts of \hat{z} and \hat{w} .

In what follows, we will use the following definition in our analysis: the spatially homogeneous equilibrium points \mathbf{K}_∞ and \mathbf{L}_∞ are (linearly) stable if $\text{Re}\{\sigma\} < 0$, i.e., if all modes go to zero as $t \rightarrow \infty$.

Proposition 1 - The equilibrium points K_∞ and L_∞ of (9a)-(9d) are unstable if and only if the following dispersion relation is satisfied:

$$c(c - zb) \geq z^2 w \quad (15)$$

where b , c , z and w are given in (14a)-(14b).

Proof: The two roots of (13) are given by:

$$\sigma_1 = \frac{1}{2} \left(-\hat{z} + \sqrt{\hat{z}^2 - 4\hat{w}} \right) \text{ e } \sigma_2 = \frac{1}{2} \left(-\hat{z} - \sqrt{\hat{z}^2 - 4\hat{w}} \right).$$

whose real parts are:

$$\text{Re}\{\sigma_1\} = \frac{1}{2} \left(-z + \sqrt{\frac{\sqrt{e^2 + f^2} + e}{2}} \right) \text{ e } \text{Re}\{\sigma_2\} = \frac{1}{2} \left(-z - \sqrt{\frac{\sqrt{e^2 + f^2} + e}{2}} \right),$$

being $e = z^2 - b^2 - 4w$ and $f = 2zb - 4c$. Since $\text{Re}\{\sigma_2\}$ is always negative, K_∞ and L_∞ will be unstable if and only if $\text{Re}\{\sigma_1\} \geq 0$, that is, if and only if:

$$\begin{aligned} & \sqrt{\frac{\sqrt{e^2 + f^2} + e}{2}} \geq z \\ & \Leftrightarrow e^2 + f^2 \geq (2z^2 - e)^2 \\ & \Leftrightarrow [z^2 - (b^2 + 4w)]^2 + (2zb - 4c)^2 \geq (z^2 + b^2 + 4w)^2 \\ & \Leftrightarrow (2zb - 4c)^2 \geq 4z^2(b^2 + 4w) \\ & \therefore c(c - zb) \geq z^2 w \quad \square \end{aligned}$$

Corollary 1 - Consider a positive transport cost $\rho > 0$. If the equilibrium points K_∞ and L_∞ are unstable, then $w \leq 0$.

Proof - Remembering the definitions of b , c , z and w , given in (14a)-(14b), and that K_∞ and L_∞ are unstable if and only if inequality (15) is satisfied, it follows that:

$$-\rho^2 v \geq z^2 w \quad (16)$$

where:

$$z = (1 + d)k^2 + \beta(1 - \phi) + 1, \quad (17a)$$

$$w = dk^4 + [1 + \beta(1 - \phi)(d - \chi)]k^2 + \beta(1 - \phi), \quad (17b)$$

$$v = dk^6 + [1 + \beta(1 - \phi)d]k^4 + \beta(1 - \phi)k^2. \quad (17c)$$

Since $v \geq 0$, the only way (16) can be satisfied is if $w \leq 0$ \square

The following corollary and note are presented in Juchem Neto and Claeysen (2014).

Corollary 2 - There exists a critical value for the capital-induced labor migration, $\bar{\chi}$, such that $\chi \geq \bar{\chi}$ implies the existence of an interval of wavenumbers (k_1, k_2) where $w = w(k) \leq 0$. Besides of that, $\bar{\chi}$ is given by:

$$\bar{\chi} = \left(\sqrt{\frac{1}{\beta(1 - \phi)}} + \sqrt{d} \right)^2. \quad (18)$$

Note - If $\rho = 0$ we have that $\chi \geq \bar{\chi}$ is a necessary and sufficient condition for K_∞ and L_∞ to be unstable. That is, if there is no capital transport cost, there exists a critical value for the intensity of capital-induced labor migration, $\bar{\chi}$, such that, if $\chi < \bar{\chi}$, the spatially homogeneous equilibrium points K_∞ and L_∞ are stable, and if $\chi \geq \bar{\chi}$, there is an interval of wavenumbers $[k_1, k_2]$ where $w = w(k) \leq 0$, and therefore the equilibrium points are unstable. In terms of the original parameters, the condition for instability is give by:

$$\chi_L \geq b \left(\frac{\delta^\phi}{sA} \right)^{\frac{1}{1-\phi}} \left(\sqrt{\frac{\delta d_L}{a}} + \sqrt{\frac{d_K}{1-\phi}} \right)^2 =: \chi_c \quad \square \quad (19)$$

Now we present the main result of this paper:

Proposition 3 - Consider $\chi \geq \bar{\chi}$. Then, there exists a critical value for the capital transport cost, $\bar{\rho}$, such that, if $\rho \leq \bar{\rho}$, the spatially homogeneous equilibrium points K_∞ and L_∞ are unstable, and if $\rho > \bar{\rho}$, they are stable. Besides of that, this critical value is given by:

$$\bar{\rho} = \max_{k \in [k_1, k_2]} \sqrt{-\frac{z^2 w}{v}}, \quad (20)$$

where z , w and v are as in (17a)-(17c).

Proof - If $\chi > \bar{\chi}$, by Corollary 2 we have that $w = w(k) \leq 0$ for some interval of wavenumbers $k \in [k_1, k_2]$, with $0 < k_1 < k_2$. Then we can write (16) as $\rho \leq \Phi(k)$, where $\Phi(k)$ is defined as $\Phi(k) = \sqrt{-\frac{z^2 w}{v}}$, being $z = z(k)$, $w = w(k)$ and $v = v(k)$ given by (17a), (17b) and (17c), respectively. Since $\Phi(k)$ is a strictly concave function, it has an unique global maximum at some point $k^* \in (k_1, k_2)$. In addition, since $\Phi(k) \geq 0 \forall k$, such a maximum value $\Phi(k^*)$ must be positive. Therefore, if $0 \leq \rho \leq \Phi(k^*)$ the system

presents unstable modes, and the result follows \square

Considering the original dimensional parameters, this implies that the critical value for the physical capital transport cost is given by $\rho_c = \sqrt{\frac{a}{d_K}} \bar{\rho}$, provided $\chi_L \geq \chi_c$, such as in (19). Then, if $\rho_K \leq \rho_c$, the spatially homogeneous equilibrium are unstable, and if $\rho_K > \rho_c$, they are stable.

This is an interesting result, since it reproduces in a macroeconomic spatial growth model, a fundamental result obtained from the core-periphery model of the New Economic Geography: that there is a critical value for the transport cost above which the economy converges to a homogeneous state, and if this cost is small enough (below this critical value), the economy shows the formation of clusters (and, in the model proposed here, the development of cycles).

4 Numerical Simulations

In order to verify the spatio-temporal dynamics generated by the model and the results obtained by the linear stability analysis presented in the previous section, we will carry out numerical simulations of system (9a)-(9d). We used a standard explicit finite-difference numerical scheme in the implementation of these simulations (see Tveito and Winther (1998), for example), and considered the following parameters:

$$a = 0.02, \quad b = 0.01, \quad \delta = 0.05, \quad s = 0.2, \quad A = 1, \quad \phi = 0.5,$$

which imply in the adimensional parameters $\beta = 2.5$ and $d = 1$.

With these parameters, we get the critical value for the capital-induced labor migration of $\bar{\chi} = \frac{9}{5} + \frac{4}{\sqrt{5}} \approx 3.59$ (see equation (18)). Besides of that, in the simulations below we set the size of the economy as $l = 200$ and consider the following initial conditions:

$$K(0, x) = 1, \quad L(0, x) = 0.1e^{\frac{(x-100)^2}{49}}.$$

In this way, we can avoid the influence of the boundary in the evolution of the initial conditions (provided we do not consider large periods of time in the simulations), since it is not possible to simulate an unbounded economy. As we comment in the conclusions, we let the analysis of a bounded economy for a future work.

Considering the parameters above, in Figure 1 (a) we plot the bifurcation curves $\rho = \Phi(k)$ for three different scenarios: $\chi = 3 < \bar{\chi}$ (stable, with the economy always converging to the spatially homogeneous equilibrium $K_\infty = L_\infty = 1$), $\chi = 5 > \bar{\chi}$ (showing an interval of unstable modes when transport cost ρ is smaller than the maximum $\Phi(k^*)$), and $\chi = 10 > \bar{\chi}$ (also showing an interval of unstable modes, but larger than the previous one, when we keeping the cost constant). In (b) we present the critical values for the adimensional transport cost parameter (20), $\bar{\rho}$, as a function of the capital-induced labor migration coefficient χ . Note that we only have a critical value $\bar{\rho}$ when $\chi \geq 3.59 \approx \bar{\chi}$, and that this critical value increases when χ increases. In (c) and (d) we present graphs similars to that in (a) and (b), but considering the original dimensional transport cost in

physical terms, ρ .

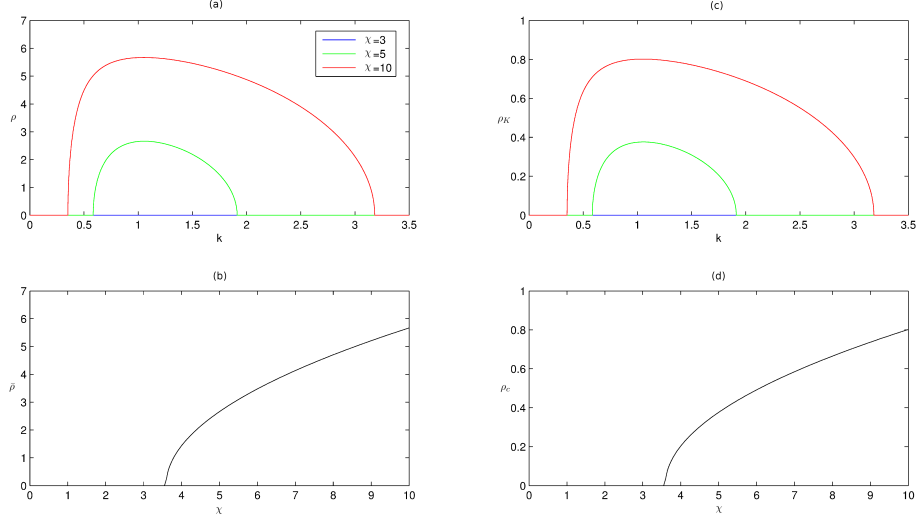


Figure 1: Bifurcation Curves

In Figure 2 we present numerical simulations showing the spatio-temporal evolution for the densities of capital (first row) and labor (second row) in the economy. In this scenario we keep the intensity of the capital-induced labor migration fixed at $\chi = 5 > \bar{\chi}$, and vary the transport cost coefficient ρ , from 0 to 3. For this χ , the critical value for the transport cost is given by $\bar{\rho} \approx 2.66$. Then, the spatially homogeneous equilibrium will be unstable only if $\rho \leq \bar{\rho} \approx 2.66$. As we can see in this figure, for $\rho = 3 > \bar{\rho}$, the economy converges to the homogeneous state of the system $K_\infty = L_\infty = 1$; for $0 \leq \rho \leq 2.5 < \bar{\rho}$, there is formation of capital and labor agglomerations. For $\rho = 0$, these clusters are stationary and spatially symmetrical, and for $0 < \rho \leq 2.5$ they are moving to the economy left boundary, case in which we have a break of symmetry.

In Figure 3 we show the same simulations as in Figure 2, but considering $\chi = 10$, case in which we will have instability only if $\rho \leq \bar{\rho} \approx 5.67$. As we can see, if $\rho = 6 > \bar{\rho}$, the economy converges to the homogeneous state; and if $0 \leq \rho \leq 5 < \bar{\rho}$ the economy develops unstable agglomerations and spatio-temporal cycles. Again, if $\rho = 0$ the economy shows spatial symmetry, but if $0 < \rho \leq 5$ all the spatial structure of the economy starts to move to the left.

This break of symmetry in the generated spatial patterns, when there is a positive transport cost, is consistent with the hypothesis that trade only occurs in the positive x-axis direction. When we impose such a positive transport cost, it is expected that trade to the right will decrease, while the capital and labor agglomerations will tend to concentrate in the left locations of the economy.

5 Conclusions

In this work we introduced capital transport cost in the spatial Solow model with capital-induced labor migration considered in Juchem Neto and Claeyssen (2014). Through a linear stability analysis of the spatially homogenous equilibrium of the model (which can be easily extended for a bidimensional economy), we derived conditions for which the model enters an unstable regime where it can develop economic agglomerations and cycles. In particular, we derived a critical value for the transport cost such that, if the cost is smaller than this critical value, the economy may develop agglomerations and cycles (provided the capital-induced labor migration is intense enough), and if it is bigger than this critical value, the economy converges to a spatially homogeneous steady-state, with all locations having the same density of capital and labor force.

In addition, we confirmed our linear stability analysis through numerical simulations of the model. We also observed that the introduction of a positive transport cost in the model causes a break in the symmetry of the generated spatial patterns (stable agglomerations and cycles), with a tendency for the agglomerations to concentrate in the left part of the economy. Such behavior is consistent with the hypothesis that trade only occurs in the positive x-axis direction.

In this way, with a macroeconomic model, following the approach of Regional Sciences, we reproduced a fundamental result from the core-periphery model of New Economic Geography: that economic agglomerations can occur only if the transport cost is lower than a certain critical value.

Future research may consider this model in a bounded domain, and add congestion costs to the model. Preliminary results showed that if the economy is bounded, the main feature of this model is preserved, i.e., the transport cost must be lower than a certain value for the model to develop clusters and cycles, although this critical transport cost value seems to be smaller in a bounded domain, when compared with the unbounded one presented in this paper.

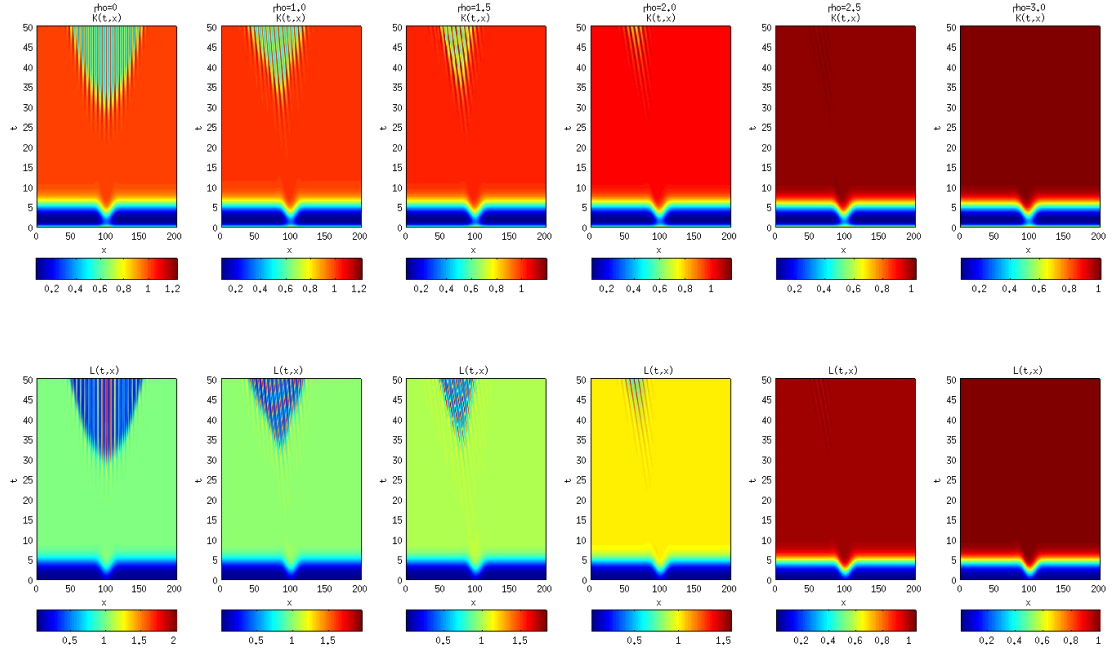


Figure 2: Spatio-temporal evolution of $L(t, x)$ and $K(t, x)$ as a function of ρ ($\chi = 5$)

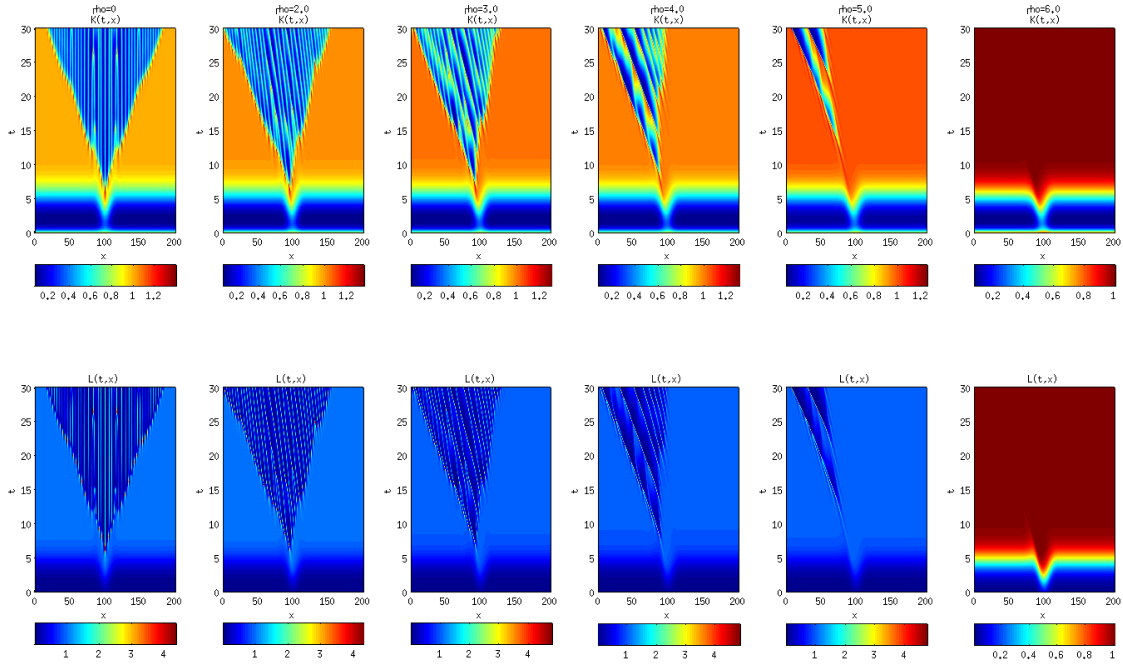


Figure 3: Spatio-temporal evolution of $L(t, x)$ and $K(t, x)$ as a function of ρ ($\chi = 10$)

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